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RELATION BETWEEN THE PETCH 'FRICTION' STRESS AND THE THERMAL ACTIVATION RATE EQUATION

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The following discussion is addressed towards establishing a connection between two alternative constitutive equations that have been used in the past to describe the temperature (and strain rate) dependence of the yield stress of iron.

Heslop and Petch[1] proposed that the temperature dependence of the yield stress of iron was primarily determined by the intrinsic lattice resistance to crystal dislocation movement, i.e. the Peierls-Nabarro stress. Their experimental measurements for this "friction" stress, $\sigma_{o_y}^{\dagger}$, were expressed in one form as

$$\sigma_{o_y}^{\dagger} = B \exp(-\beta T) \qquad (1)$$

where T is the absolute temperature and B and β are experimental constants. It was pointed out that the strain rate has a large effect on $\sigma_{o_y}^{\dagger}$ and this influence enters equation (1) implicitly through the parameter β[2,3]. However, for a fixed strain rate, (1) is relatively easy to evaluate and it may be used as well to describe measurements obtained for other materials[4,5].

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Conrad[6] has shown that the results from a considerable number of studies of the plastic yielding of iron and steel may be expressed in the relationship

$$\sigma^{\dagger}_{o_y} = \frac{2U_o}{V} + \frac{2kT}{V} \ln \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^{*}_{o}}\right) \qquad (2)$$

where $\dot{\varepsilon}$ is the tensile strain rate, k is Boltzmann's constant, and U_o, V, and $\dot{\varepsilon}^{*}_{o}$ are parameters employed in the thermal activation rate analysis. The parameters employed in equation (2) have some direct theoretical basis, e.g. U_o is an activation energy associated with the rate controlling process involved in dislocation movement, V is the activation volume through which work is done and $\dot{\varepsilon}^{*}_{o}$ is a product of several factors: a geometric factor (relating tensile strain rate and shear strain rate), the dislocation density, the area swept out in dislocation movement between obstacles, and the vibrational frequency of the dislocation line. Experiments have shown that V is itself a function of, at least, $\sigma^{\dagger}_{o_y}$, and this seems reasonable on the basis of dislocation theory.

To relate the parameters employed in (1) and (2), it may first be noted that at T = 0, the value of B is directly obtained as

$$B = \frac{2U_o}{V_o} \qquad (3)$$

where V_o is the value of V at T = 0. For iron, $U_o \approx 8.8 \times 10^{-13}$ ergs, as given by Conrad[6], and $B \approx 1.8 \times 10^{10}$ dynes/cm^2, as determined from the data of Heslop and Petch[1]. Substitution of these values in (3) gives $V_o \approx 9.6 \times 10^{-23} cm^3$, and this value compares favorably with the lowest value of $V \approx 1.2 \times 10^{-22} cm^3$ estimated by Conrad.

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To evaluate β, the right hand sides of (1) and (2) are equated and the terms rearranged in the form

$$\beta = -\frac{1}{T}\ln\left[\frac{2U_o}{VB} + \frac{2kT}{VB}\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o^*}\right] \tag{4}$$

Now, based on the work of Conrad[6], typical values, at $T = 190°K$ and $\dot{\varepsilon} = 10^{-4}\text{sec}^{-1}$, for the additional parameters in (4), are $V \approx 3.4 \times 10^{-22}\text{cm}^3$ and $\dot{\varepsilon}_o^* = 5 \times 10^8 \text{sec}^{-1}$. This value of T is selected as the median temperature for the range (80-300°K) over which most measurements have been made and the value of V is also the median value obtained by Conrad for this temperature range. Using the preceding estimates, it may be seen that

$$\frac{2U_o}{VB} > 0,\ -\frac{2U_o}{VB} < \frac{2kT}{VB}\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o^*}\right) < +\infty \tag{5}$$

so that β may be expanded in series, taking into account, also

$$\left|\,4\frac{U_o}{VB}\,\right| > \left|\,\frac{2kT}{VB}\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o^*}\right)\right| \tag{6}$$

to give

$$\beta \approx \frac{k}{U_o}\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o^*}\right) - \frac{1}{T}\ln\left(\frac{2U_o}{VB}\right) \tag{7}$$

For lower or higher temperatures, or different strain rates, it occurs that the change in $\sigma_{o_y}^{\dagger}$ and, hence V is such that (7) should still hold very well. Also because V increases as the temperature increases (because of the variation of $\sigma_{o_y}^{\dagger}$), it appears that these changes may largely counteract one another in (7) to give a constant value of β.

as indicated by the experimental basis for (1). This point is further examined below by comparison between the experimental value of β taken from (1) and that derived from (7).

The value of β obtained from the experimental data of Heslop and Petch[1], at $\dot{\varepsilon} = 10^{-4}$ sec^{-1}, is 1.43×10^{-2} °K^{-1}. Employing the preceding values given for U_o and $\dot{\varepsilon}_o^*$, the value of the second term on the right hand side of (7) is obtained by difference as

$$- \frac{1}{T} \ln \left(\frac{2U_o}{VB}\right) \approx 9.7 \times 10^{-3} \ {}^\circ K^{-1} \qquad (8)$$

At 190°K, the value of (8) obtained directly from U_o, V and B is 6.5×10^{-3} °K^{-1}. Since (8) should not vary with temperature, then at fixed $\dot{\varepsilon}$,

$$\Delta \left[- \frac{1}{T} \ln \frac{2U_o}{VB} \right] = \frac{1}{T^2} \left\{ \ln \left(\frac{2U_o}{VB}\right) \right\} \Delta T + \frac{1}{TV} \Delta V \qquad (9)$$

From (9), the variation in (8) for the temperature interval (190-80)°K and (190-300)°K is $\pm 1.7 \times 10^{-3}$ °K^{-1}, respectively, using for the limiting temperatures the values of $V \approx 2.1 \times 10^{-22}$ and 4.6×10^{-22} cm^3 taken from Conrad. In light of the varied experimental data and the estimates involved in all the quantities employed, the variation given by (9) is considered to indicate that the second term on the right hand side of (7) may be approximated by a constant value, β_o. Taking $\beta_o \approx 6.5 \times 10^{-3}$ °K^{-1}, then (7) gives a value of $\beta \approx 1.1 \times 10^{-2}$ °K^{-1}, which compares favorably with the value of 1.43×10^{-2} °K^{-1} determined by Heslop and Petch[1]. Thus (1) may be finally rewritten

$$\sigma^{\dagger}_{o_y} \approx \frac{2U_o}{V_o} \exp \left[- \left\{ \beta_o + \frac{k}{U_o} \ln \left(\frac{\dot{\varepsilon}_o^*}{\dot{\varepsilon}}\right) \right\} T \right] \qquad (10)$$

Note that in a numerical evaluation of $\sigma_{o_y}^{\dagger}$ the smaller value of B determined from the thermal activation rate parameters would largely compensate for the smaller β value given above. In (10), therefore, the Petch "friction" stress has been expressed fairly directly in terms of the thermal activation rate analysis parameters. A definite connection between the equations (1) and (2) is established.

In conclusion, some brief comments should perhaps be made concerning the usefulness of the foregoing analysis. It shows an explicit influence of the strain rate on the parameter β, in agreement with the previous suggestion by Heslop and Petch[2] and Petch[3] that this parameter is strain rate dependent. The analysis offers a further indication of the relative self-consistency of the large amount of data collected until the present time on this aspect of the deformation of iron, a view promoted initially by the work of Conrad[6].

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